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IMPLEMENTATION AND PRACTICAL APPLICATION OF NON-EQUIVALENT BIAXIAL MEASUREMENTS FOR HYPERELASTIC MATERIALS

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1 Introduction

In the case of hyperelastic material models, it is not possible to reliably fit the parameters of the constitutive model to general load cases from a single set of uniaxial measurements. In order to obtain a more accurate model, different measurements are used, but these are very complex and difficult to perform in most cases.

The most commonly used measurement datasets for fitting are data from uniaxial and biaxial tensile tests and constrained uniaxial loading (planar tension), but due to the complexity of the measurement implementation, fitting is often done with less data. It can be seen that the stress-strain relationship obtained for the uniaxial and equibiaxial test cases envelops the case of planar tension from two sides, the application of which helps to define a better material model. Treloar's dataset also describes three different cases of experiments, in which the results of these tensile experiments were analysed [1]. However, if we perform biaxial measurements and can control the displacements along the two axes independently, we may be able to generate a new dataset that is neither uniaxial nor equibiaxial measurement data, this dataset describes an intermediate state. This way, we are also able to make other types of measurements in the same measurement setup, which can lead to a more accurate result when fitting a material model.

2 Details of work

In the present work, silicone specimens, which are capable of large elastic deformation, were used. These materials suffer negligible residual deformation even under high strain, and therefore, the mechanical behaviour can be well approximated by a hyperelastic material model [2]. For the measurements, we used an in-house developed biaxial material testing machine, which can be used to impose arbitrary independent displacements along the two axes using stepper motors, so that the specimen can be loaded to an arbitrary general biaxial stress state of our choice. A schematic illustration of the general biaxial in-plane loading case is shown in Fig. 1.

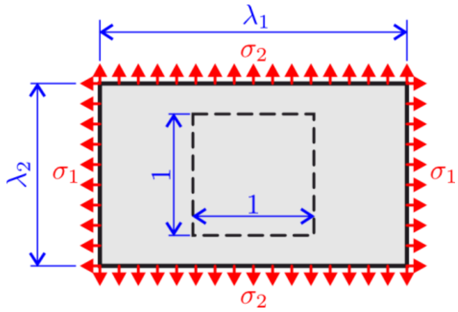

Fig. 1 Schematic representation of the general biaxial in-plane loading case.

During the measurements, the exact biaxial stress state we have prescribed is only present at the centre of the test specimen, so that the force and displacement quantities that can be easily measured physically are not sufficient for us to directly determine the numeric values we are looking for. However, knowing the load and the geometry of the specimen, it is possible to approximate the stress state using a stress distribution and a geometric factor [3,4]. A good approximation for the design and evaluation of the measurements is to assume the material to be incompressible. By imposing the strains along two perpendicular axes, we can determine the total deformation state for our load case at any instant. We can characterize a general biaxial stress case by its deformation gradient and stress tensor matrix, which can be written as

$$\boldsymbol{F} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_\mathrm{T} \end{bmatrix}, \quad \boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (1)$$

where λ_T is the transverse stretch. Note that $\lambda_1\lambda_2\lambda_T = 1$ for the incompressible case. If we impose displacement along the two axes independently, we can also align the elongations, so we are free to impose any relationship between the stretch values λ_1 and λ_2. Performing the measurements, we used a linear combination of the uniaxial and equibiaxial cases. The relationships for the principal stretches in this case are as follows:

$$\lambda_2 = (1-\alpha)\cdot\lambda_1^{-0.5} + \alpha\cdot\lambda_1. \quad (2)$$

For $\alpha = 0$, we obtain uniaxial loading, while for $\alpha = 1$, we obtain equibiaxial loading. It is important to note that in the case of planar tension loading, when using the above relationship, the stretch value in the 1-direction becomes a function of the α parameter, since for this loading mode $\lambda_2 = 1$. Solving the above equation for α yields

$$\alpha = \left(\lambda_1 + \sqrt{\lambda_1} + 1\right)^{-1}. \quad (3)$$

The resulting solution is illustrated in Fig. 2. Note that for $\lambda_1 = 1$, the value of α is 1/3, while for $\lambda_1 = 0$, it is 1.

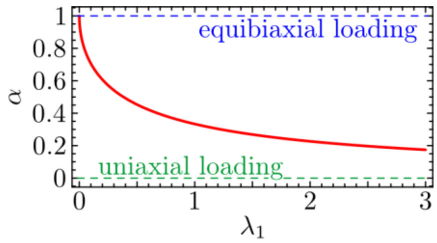

Fig. 2 Resulting values of the α parameter in the case of planar tension.

The stress solution for the general biaxial loading case can be derived analytically using the α parameter for most incompressible hyperelastic material models. This enables the parameter identification process to be carried out using closed-form expressions, facilitating the optimization task. Measurements can be performed using various values of the α parameter, thereby providing data corresponding to different loading conditions for the parameter fitting procedure.

Although the α parameter provides information about the loading mode, it does not directly indicate where the resulting stress state lies between uniaxial and equibiaxial stress conditions. To characterize the stress state, the stress triaxiality variable is a suitable choice. Stress triaxiality can be defined simply by relating the instantaneous hydrostatic stress to the instantaneous equivalent von Mises stress:

$$\eta = \frac{\sigma_h}{\sigma_{eq}^{vM}}. \quad (4)$$

This variable is widely used in damage mechanics models but can also be advantageously applied in this context.

3 Conclusions

Within the framework of this research, we have defined a general biaxial stress state, which relates two commonly used cases with continuous transitions, and given the analytical solutions for these cases in terms of strain and stress. In addition, a measurement setup suitable for the case presented previously was set up, and a series of measurements were made and evaluated, so that we were able to build a finite element model by generating useful numerical data, fitting a material model to the material, and comparing the measured results with simulations. With the newly proposed procedure, we are able to investigate the behaviour of a material in several cases without changing the experimental setup many times, which allows us to characterise the material under investigation more accurately.

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