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THE DEPENDENCE OF THE DEGREE OF NON-ADDITIVITY OF MECHANICALLY LOADED SYSTEMS ON THE LOADING RATE

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1. Introduction

Considering that the phenomenon of fracture is a manifestation of the series of non-linear processes, which characterize complex dynamical systems, it is reasonable to approach the specific phenomenon using advanced statistical tools rather than traditional Statistical Mechanics [1,2], taking into account that the response of such systems (at non-equilibrium stages) is not compatible to the thermodynamic principle of additivity.

This incompatibility is attributed to the non-independent nature of the processes responsible for fracture, which, after specific load levels tend to produce well-organized and mutually interacting networks (sub-systems) of micro-cracks. In this context, Boltzmann-Gibbs (BG) Statistical Mechanics (and the respective BG entropy concept, S_{BG}) must be properly modified to cope with such phenomena, which are characterized by long-range interactions and memory effects (in other words by non-additivity), giving birth to the relatively new discipline of Non-Extensive Statistical Mechanics (NESM). Among various NESM formulations, the one most broadly used is that introduced by Tsallis [1], based on an extended definition of entropy, as:

$$S_q = k\frac{1-\sum_{i=1}^{w} P_i^q}{q-1} \tag{1}$$

where k is Boltzmann's constant, P_i the probability of the i^{th} configuration, w the number of possible configurations, and, q the entropic index, quantifying the degree of non-additivity of the system [1].

While S_{BG}, is maximized by the Gaussian (distribution), S_q is maximized at distributions denoted as q-Gaussians, described as [2]:

$$P(\chi,\beta) = A_q e_q\left(-\beta_q \chi^2\right) \tag{2}$$

In Eq.(2) A_q and β_q are numerical constants and $e_q(z)$ stands for the q-exponential function, which is defined as follows [1, 2]:

$$e_q(z) = [1+(1-q)z]^{\frac{1}{1-q}} \tag{3}$$

Within the frame of the above argumentation, an attempt is described here to explore the potential dependence of the degree of non-additivity of a mechanically loaded system (i.e., specimens made of brittle rock) on the loading rate imposed externally, by quantifying the respective values of the q-index.

2. Materials and methods

To achieve the target of this study, advantage is taken of experimental data gathered from a protocol (described thoroughly in a previous publication [3]) comprising uniaxial compression tests with prismatic specimens made of Dionysos marble (which is used for the needs of the ongoing restoration project of the Athenian Acropolis). The acoustic activity was recorded using sensors of the R6α type. The principal characteristic of that protocol is that the loading rates imposed varied within a relatively broad interval, ranging from 34 to 350 kPa/s.

The quantity used here, to explore the self-organized state of the fracture process, is the amplitude, A, of the Acoustic Emissions, expressed in terms of the so called "*variable returns*" parameter [4, 5] (quantifying, in fact, the fluctuations of the amplitudes), defined as:

$$(\delta A)_i = A(t_{i-1}) - A(t_i) \quad (4)$$

In Eq.(4) $A(t_i)$ is the amplitude of the signal that was recorded at the instant t_i. Usually, $(\delta A)_i$ is normalized over the variance σ, as:

$$(\Delta A)_i = \frac{(\delta A)_i - \mu}{\sigma} \quad (5)$$

where μ is the respective mean value.

As a next step, the interval of the values of $(\Delta A)_i$, namely the interval $[(\Delta A)_{min}, (\Delta A)_{max}]$, is divided into a number of sub-intervals of equal width. For each sub-interval the Probability Density Function (PDF) of the distribution of $(\Delta A)_i$ is determined in terms of the mean value δ of each one of the sub-intervals. Combining Eqs.(2) and (3) the PDF reads as:

$$PDF(\delta, \beta_q) = A_q\left[1-(1-q)\beta_q\delta^2\right]^{\frac{1}{1-q}} \quad (6)$$

3. Results, discussion and conclusions

The experimental data are then fitted by means of Eq.(6) using commercial available software. A very good correlation is revealed, as it can be clearly seen from Fig.1, in which the q-Gaussian distributions are plotted for the four classes of loading rates imposed in the specific experimental protocol, (they were equal to 34, 84, 140 and 350 kPa/s).

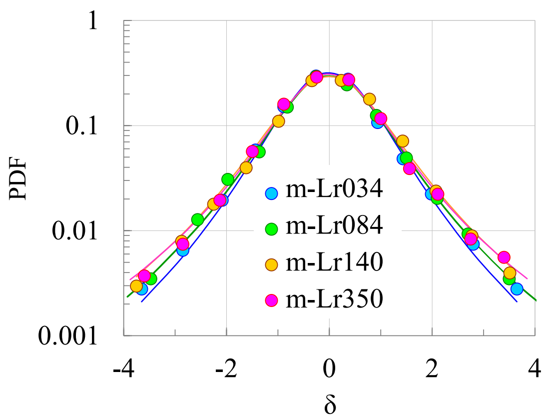

Fig. 1. The q-Gaussian distributions for the four classes of specimens.

The respective values of q, as obtained from the fitting procedure, are plotted in Fig.2, against the loading rate imposed. It is clearly seen that a monotonously increasing trend characterizes the response of q, suggesting, that the degree of non-additivity of the fracture process is an increasing function of the loading rate. The dependence of q on the loading rate seems to be governed by a power law (see Fig. 2), however additional protocols are necessary before definite conclusions are drawn.

A similar, positive correlation of the displacement rate (rather than of the loading rate) with q, was highlighted recently by Shan et al. [5], in terms of the evolution of the electric potential, for specimens made of various rocks. In that study, the potential use of q as pre-failure index was, also, suggested by the authors.

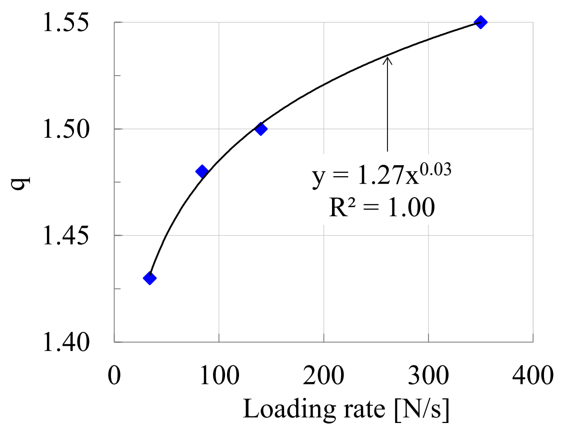

Fig. 2. The dependence of the entropic index q on the loading rate.

Concluding, it could be stated that increasing the loading/displacement rate the damage mechanisms activated within the bulk of a mechanically loaded system are intensified, leading to denser networks (subsystems) of micro-cracks, or, equivalently to further deviation of the system (and the respective processes) from additivity.

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