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CONSTITUTIVE MODELLING OF POLYMER FOAMS USING AN ASYMMETRIC POISSON'S RATIO

Márton KAMMERER[1], Attila KOSSA[2]

[1] Department of Applied Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, Műegyetem rkp. 3., Budapest, 1111, Hungary, E-mail: kammerer.marton@edu.bme.hu

[2] 0000-0003-3638-3237, Department of Applied Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, Műegyetem rkp. 3., Budapest, 1111, Hungary, E-mail: kossa@mm.bme.hu

1. Introduction

Open-cell polymeric foams exhibit a pronounced asymmetry when subjected to uniaxial tension and compression: the axial stress–strain curves and the accompanying lateral (transverse) strains differ drastically. This behavior can be traced back to the nonlinearity of the polymer matrix material coupled with the complex geometry of the cellular microstructure.

Hyperelastic constitutive models are the standard choice for modeling the elastic response of such materials. Yet a review of the literature reveals that even the most complex ones of the existing models [1] invariably fail to reproduce the dual character of tension-compression characteristics, particularly the lateral strains [2]. To overcome this limitation and capture the experimentally observed asymmetry we introduce a novel hyperelastic constitutive model asymmetric in Poisson's ratio.

2. Materials and methodology

The compressible Ogden–Hill hyperelastic constitutive model [1] is the prevailing model for describing polymeric foams, for which the strain energy density function is defined as:

$$W(\lambda_1,\lambda_2,\lambda_3) = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 + \frac{1}{\beta_i}\left(J^{-\alpha_i\beta_i} - 1\right)\right), \tag{1}$$

where λ_k $(k = 1,2,3)$ denote the principal stretches, J the volume ratio and N is the order of the material model. In ABAQUS, this model appears under the name Hyperfoam, and defines the β_i parameters as:

$$\beta_i = \frac{\nu_i}{1-2\nu_i}. \tag{2}$$

In some special cases, when $\nu_i = \nu$, this parameter can be associated with Poisson's ratio.

Building on this identity, we introduce an asymmetric variant where we use J to switch between "tension" $(J > 1)$ and "compression" $(J < 1)$ like cases and for these we use different Poisson's ratios:

$$W(\lambda_1,\lambda_2,\lambda_3) = \begin{cases} \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 - \alpha_i \ln(J)\right), & J < 1, \\ \frac{2\mu}{\alpha^2}\left(\lambda_1^{\alpha} + \lambda_2^{\alpha} + \lambda_3^{\alpha} - 3 + \frac{1}{\beta}\left(J^{-\alpha\beta} - 1\right)\right), & J > 1, \\ \frac{1}{2}\left(W^{c}(\lambda_1,\lambda_2,\lambda_3) + W^{t}(\lambda_1,\lambda_2,\lambda_3)\right), & J = 1. \end{cases} \tag{3}$$

The "tension" part adopts a first-order Hyperfoam model, whereas the "compression" part is constrained to zero Poisson's ratio, a widely used approximation for open-cell polymeric foams [3].

Because uniaxial tension and compression tests provide the primary calibration data, it is convenient to write the corresponding 1st Piola–Kirchhoff stress in closed form, allowing direct parameter fitting against experimental curves.

$$P_1 = \frac{\partial W}{\partial \lambda_1} = \begin{cases} \sum_{i=1}^{N} \frac{2}{\lambda_1}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} - 1\right), & J < 1, \\ \frac{2}{\lambda_1}\frac{\mu}{\alpha}\left(\lambda_1^{\alpha} - \lambda_1^{-\alpha\nu}\right), & J > 1, \\ \frac{1}{2}\left(P_1^{c} + P_1^{t}\right), & J = 1. \end{cases} \tag{4}$$

The proposed model was calibrated and validated against uniaxial tension- and compression-test data obtained for an open-cell polymeric foam [4]. As illustrated in Fig. 1, the predicted stress–strain response reproduces the experimental curves with great accuracy.

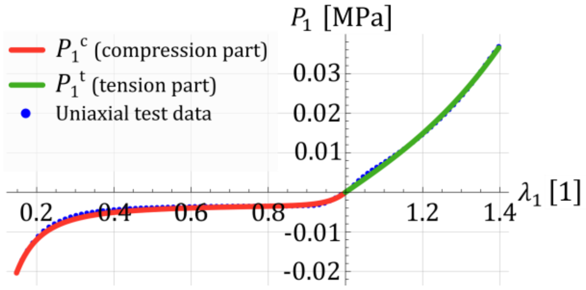

Fig. 1. Fitting on uniaxial test data.

Transverse strains were extracted from the uniaxial tests by post-processing recorded videos with a binarization technique. The measured trends confirm that Poisson's ratio of 0 is adequate in compression, while in tension incompressibility is the best approximation – an observation that was not anticipated. Consequently, the tensile branch of the Hyperfoam formulation collapses to the first-order, incompressible Ogden model.

For model validation we constructed a large-scale bending apparatus (Fig. 2), on which finite strain configurations can be achieved in discrete steps. The obtained reaction forces and geometrical datasets were later compared to FEM predictions.

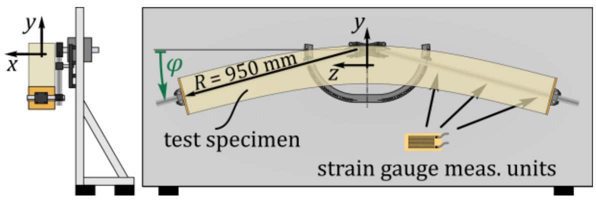

Fig. 2. Design of the large-scale bending apparatus.

All simulations were conducted in ABAQUS [5]. As a first approximation to emulate the proposed asymmetric constitutive law, the beam was partitioned into tensile- and compressive-dominant regions (Fig. 3), constructing the separated FEM model. Each region was assigned a Hyperfoam material definition with its own calibrated parameter set.

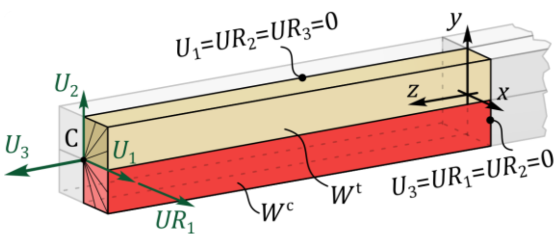

Fig. 3. The separated FEM model.

3. Conclusions

Simulation results indicate that in bending (where tension and compression arise simultaneously), the separated FEM model achieves reaction predictions on par with conventional Hyperfoam fits yet captures transverse strains with far greater accuracy (Fig. 4).

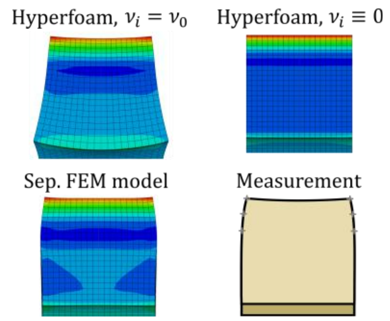

Fig. 4. Cross sectional view of the beam at $\varphi = 30°$ for multiple cases.

The collected evidence shows unambiguously that, in bending, the asymmetric hyperelastic formulation yields better results at transverse strains than its Hyperfoam counterpart. Motivated by this performance, we have coded dedicated UHYPER and UMAT subroutines for ABAQUS, providing a platform for complete studies and more complex loading scenarios. Future work will address numerical implementations of hyperelastic models that couple volumetric and isochoric behavior, including the present formulation.

4. Acknowledgements

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