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EFFECTS OF STEERING ANGLE SATURATION ON STABILITY OF REVERSING TRAILERS

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1. Introduction

Driver assistance systems are now a standard feature in modern passenger vehicles, primarily for improving safety and reducing fuel consumption. Extending these systems to freight transport is essential, particularly when addressing one of the most challenging maneuvers, the reversing. Reversing a long, articulated truck–semitrailer combination is inherently unstable, and requires active control to ensure safe maneuvering [1, 2].

The effect of steering angle saturation changes the dynamical properties of a truck–semitrailer combination. In this study, we examine the dynamics of a truck–semitrailer reversing maneuver and analyze how steering angle saturation affects the stability of the autonomous feature proposed for reversing a truck–semitrailer.

2. Mechanical model

To represent the dynamics of the articulated vehicle, a kinematic model with rigid wheels is adopted. The longitudinal speed V of the towing vehicle's rear axle is assumed to remain constant. This assumption is accepted in the literature, particularly for low-speed maneuvers. The configuration is depicted in Fig. 1. The truck and the trailer are represented as two rigid rods connecting the midpoint of the front F and rear axles R of the truck, and the hitch point K and axle T of the trailer, giving the so-called single-track model.

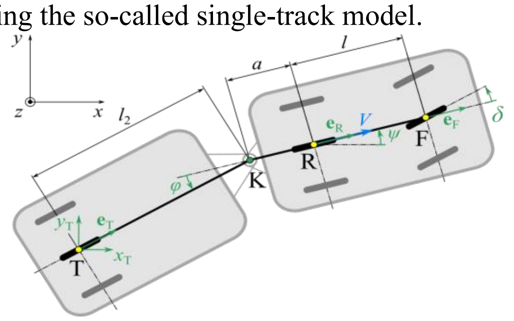

Fig. 1. Mechanical model of the truck–semitrailer

The vehicle combination is actuated exclusively through the steering angle δ of the truck's front axle. The motion is described by the vector of state variables: $\mathbf{x} = [x_\mathrm{R}\ y_\mathrm{R}\ \psi\ \varphi\ \delta\ \omega]^\mathrm{T}$, where x_R and y_R denote the positions of the semitrailer's axle T in the global coordinate system, ψ is the yaw angle of the truck, and φ is the hitch angle of the trailer. Variables δ and ω are introduced in order to consider the dynamics of the steering mechanism as a PD-controlled one degree-of-freedom system: δ is the steering angle, ω is the steering rate. The governing equations are derived from the kinematic constraints:

$$\dot{x}_\mathrm{R} = V\cos\psi\,, \quad \dot{y}_\mathrm{R} = V\sin\psi\,, \quad \dot{\psi} = \frac{V}{l}\tan\delta, \quad (1)$$

$$\dot{\varphi} = -\frac{V}{l l_2}(l\,\sin\varphi + (l_2 + a\,\cos\varphi)\,\tan\delta), \quad (2)$$

and from the dynamics of the steering mechanism:

$$\dot{\delta} = \omega, \quad \dot{\omega} = -p(\delta - \delta_\mathrm{des}) - d\omega, \quad (3)$$

where p and d are the gains of the lower-level steering controller.

3. Steering angle saturation

The higher-level controller's duty is to stabilize the motion along the x-axis by means of the control law providing the desired steering angle

$$\delta_\mathrm{des}(t) = \mathrm{sat}(-P_y y_\mathrm{R}(t-\tau) - P_\psi\psi(t-\tau) - P_\varphi\varphi(t-\tau)), \quad (4)$$

where P_y, P_ψ and P_φ are the control gains. The steering angle of the vehicle has a physical limitation. This phenomenon is considered in our model by the saturation function, which is smoothed near the limitation based on [3]. Saturating the steering angle changes the dynamics of the vehicle combination fundamentally by causing another source of nonlinearities in the system.

Time delay τ is also considered in the control loop, which has relevancy not only for human drivers (reactions time), but also for autonomous vehicles (e.g. image processing and computation time).

4. Results and validation

Bifurcation diagram representing the nonlinear effects of the steering angle saturation and considering the time delay in the control loop was conducted by numerical continuation using the Matlab package DDE-Biftool [4]. The results are validated on a small-scale test rig, shown in Fig. 2. The sensor setup provides high-precision position and angle data at 1 kHz sampling frequency.

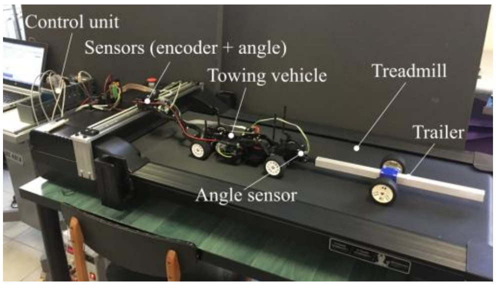

Fig. 2. Experimental setup

Both theoretical and experimental results are shown in Fig. 3 via a bifurcation diagram in the plane of the control gain P_ψ and the amplitude of the hitch angle φ. Gains related to linearly stable or unstable motion (i.e., zero amplitude) are denoted by green and red lines, respectively. The results of the theoretical calculations (solid lines) show an unstable limit cycle above the linearly stable region for the applied δ^{sat} saturation value at $P_\psi = 6$, meaning a subcritical Hopf-bifurcation. However, due to the steering angle saturation, the bifurcation branch folds back and turns into stable limit cycle. This phenomenon also infers the appearance of a bistable region, where the actually realized motion depends on the magnitude of the perturbation. In the high control gain region, a stable limit cycle is situated above the linearly unstable region. The experimental results (dashed lines) show good agreement with the theory. Amplitudes of the stable limit cycle match beyond expectations, and the fold point at the high gain region, when stable limit cycle turns into unstable, is also detected (amplitudes blow up). The phenomenon of hysteresis appears as relevant jumps in the amplitudes in the linearly stable cases (low gain region), however, small amplitudes (~3°) remain there due to noises and other inaccuracies in the experimental setup.

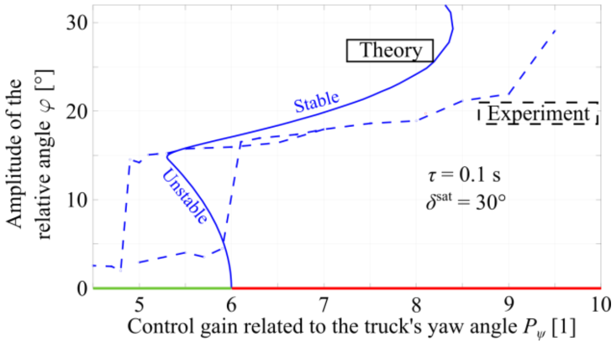

Fig. 3. Bifurcation diagram and its validation

5. Conclusions

In this paper, the rectilinear reversing of a truck–semitrailer was investigated using theoretical and experimental methods. A formerly designed controller, suitable for reversing the articulated vehicle was supplemented with saturation on the steering angle. The modified dynamics and the nonlinear behavior were revealed with bifurcation analysis and was verified with small-scale experimental results. This study has relevance in industry in assisting truck drivers while ensuring stability during maneuvering in reverse.

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