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KINEMATIC ANALYSIS OF THE COMPLEX PLANETARY GEAR TRAIN OVERVIEW

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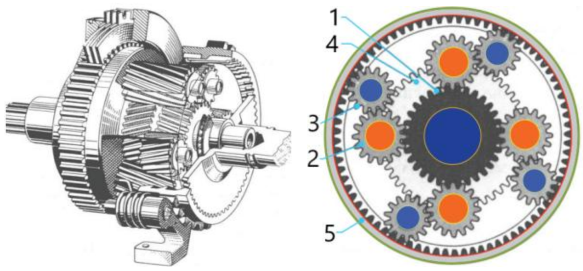
1. Introduction

Planetary gear trains represent a special type of gear transmission whose element motion resembles the movement of planets around the Sun. Thanks to their compact design and the ability to achieve high transmission ratios with a relatively low total mass, they have found application in various fields of mechanical engineering.

The basic planetary gear train (PGT) consists of two central gears (one with external teeth and the other with internal teeth), planet gears, and a carrier on which the planets are mounted. By connecting the appropriate elements to the input and output shafts, as well as by fixing certain components, different configurations or planetary gear train schemes can be obtained. This is described in the literature [1] and [2]. By adding gears and carriers to the structure of a simple planetary gear train, complex planetary gear trains are formed. Their advantage over simple ones lies in achieving a greater number of different transmission ratios. This has defined the wide application of PGT in automatic transmissions of various passenger and heavy-duty vehicles, as well as buses and construction machinery.

To determine the transmission ratios, or the speeds of a multi-stage planetary gear train, it is necessary to understand the kinematics of the coupling between its elements. Considering the complexity of the motion, this represents a very challenging task. In books [1, 2], the most commonly used methods for solving the kinematics of PGT are presented.

The applying of these methods for determining the transmission ratios of a complex PGT requires a great deal of time due to the increasing complexity of the motion equations and the difficulty in expressing the transmission ratios caused by the large number of degrees of freedom. Because of these shortcomings, an overview of research related to new methods for determining the speeds of the complex Ravigneaux planetary gear train (RPGT) model is presented, Figure 1-a.



a) b)

Fig. 1. Ravigneaux planetary gear train [3]

The Ravigneaux model features a compact design with a single planet carrier, three central gears (1), (4), and (5), and two planet gears (2) and (3) that are meshed with each other, Figure 1-b. By connecting the central gear (5) to the output shaft and fixing or connecting the remaining three central elements, gears (1) and (4) and the carrier to the input shaft, it is possible to achieve the max. value of six different transmission ratios. In practice, however, due to the way the aforementioned elements are connected to the components of an automatic transmission, it is possible to achieve the max. of four forward speeds and one reverse speed.

Some of the commonly used transmissions with the RPGT are: Ford AOD, ZF 4HP14, ZF 6HP19/26, Mercedes-Benz 5G-Tronic, A5S310Z for BMW 530i, Mercedes-Benz 7G-Tronic, etc.

Taking into account the complexity of the motion regarding the elements of the mentioned multi-stage RPGT, this paper will provide a systematic review of methods, emphasizing their advantages, disadvantages, and areas of applying.

2. Kinematic Analysis

Among the standard methods, the most widely used are Willis's equation and the general motion equation in the form of a combined method. These methods are described in detail in books [1, 2]. Using the conditions given in [1], inside [4] analytically expressed the overall transmission ratio for one of the gears of the RPGT as a function of the number of teeth of the central gears. To determine the transmission ratio, two general equations were used due to the division of the RPGT into two simple planetary gear trains.

In the following text, the methods for determining kinematics of the RPGT are presented.

2.1 Lever Analogy Method

Benford and Leising [5] introduced this method as a new tool for analyzing automatic transmission systems. The method is based on drawing a vertical line with corresponding points. This line represents the lever, while the points define the central elements of the RPGT. The construction of the lever for the RPGT is shown in Figure 2.

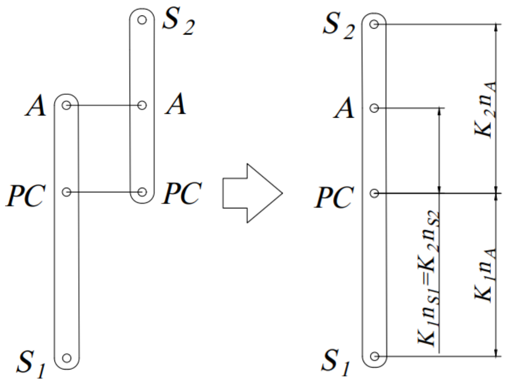


Fig. 2. Lever construction for the RPGT [8]

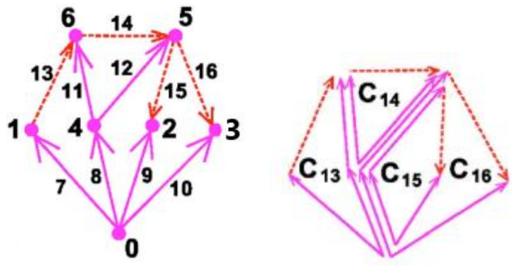
The RPGT represents a combination of two simple planetary gear trains in which the planet carrier (PC) and the central gear with internal teeth (A) are identical, while the central gears with external teeth (S1) and (S2) differ. By connecting their all levers, one lever for the analyzed RPGT is obtained, Figure 2.

On the obtained lever, the distances are then defined as $K \cdot n_A$ and $K \cdot n_S$ between the points, which represent the rotational speeds of the central gears multiplied by constants K_1 and K_2. By knowing the rotational speed of the input element and fixing the corresponding gear, the scale constants can be determined based on this relationship, and using them, the rotational speeds of all elements can be calculated. Based on the obtained values of the rotational speeds for the input and output elements, the overall transmission ratio for the corresponding speed of the RPGT can be easily determined.

Today the method applying allows determining the values of loads that occur at the gear contacts. The drawback of the method is that it does not take into account data related to the planet gears and their mutual relationships. This problem was addressed in [6] by adding nodal points that define the planet gears.

2.2 Matrix Method Based on Graph Theory

Kinematic relationships between the elements inside RPGT can be represented with graph theory. The graph construction for the RPGT, presented in literature [7], can be seen in Figure 3-a.



a) b)

Fig. 3. Construction a)-graph and b)-of the closed loop for RPGT [7]

Node of the input shaft (0) is connected to the nodes of the central gears (1) and (2) and the planet carrier (4). The carrier node (4) is connected to the nodes of the planet gears (5) and (6). The node of the central gear with internal teeth is connected to the output shaft node, which is labeled as (0). Solid connections (7, 8, 9, 10, 11, and 12) are defined with a solid line between the nodes, while transmission connections (13, 14, 15, and 16) are represented with a dashed line. Based on its construction, closed loops can be obtained (C_{13}), (C_{14}), (C_{15}) i (C_{16}) which are shown in Figure 3-b.

For the graph in Figure 3-a, the incidence matrix is determined $\Gamma^o = m \times k$ (m – nodes, k – edges) the incidence matrix, which can be reduced so that the node is not included (0). Reduced matrix Γ can be divided into two parts from which the path matrix is determined Z oriented graph. From this matrix, the cycle metric can be determined as the function of closed loops, Figure 3-b, and through it, the linear equation of the absolute angular speed can be obtained in the following form [7]:

$$\begin{bmatrix} -i_{13} & 0 & 0 & i_{13}+1 & 0 & -1 \\ 0 & 0 & 0 & i_{13}+1 & -1 & -i_{14} \\ 0 & -i_{15} & 0 & i_{13}+1 & -1 & 0 \\ 0 & 0 & 1 & i_{13}+1 & -i_{16} & 0 \end{bmatrix} \cdot \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (1)$$

Where are: i_{13}, i_{14}, i_{15} i i_{16} partial transmission ratios between the elements of the RPGT, and ω_i angular speeds of all elements. By knowing the input parameters, the matrix equation (1) is simplified. By connecting and fixing the appropriate elements of the complex PGT, the angular speeds can be expressed in matrix form ω_i, and through them, the overall transmission ratios for each of its speeds.

The procedure for determining transmission ratios using the linear equation of absolute angular speed is presented in [8]. Unlike the previous method, this study performs a structural decomposition, where the RPGT is divided into two parts, graphically represented using a rotation graph. The rotation graph is similar to the graph in Figure 3-a. The difference is that the nodal points are connected in the form of a closed polygon. Discretization of the rotation graph into two substructures, the node matrix is determined, from which the angular speed matrix is obtained. Based on the angular speed matrix, the linear equation of angular speeds in matrix form is derived, which is similar to equation (1). The difference lies in the definition of partial transmission ratios on the left side of the equation. Solving the resulting linear equation is similar to the procedure in [7].

2.3 Nomogram Method

A new method for analyzing kinematic quantities, as well as torques on the elements of PGT, was developed by [9, 10], where is RPGT presented graphically using a nomogram. The construction of the nomogram involves drawing vertical lines corresponding to the angular speeds of the PGT elements relative to the horizontal axis.

In [9, 10], a detailed procedure for obtaining speeds in an automatic transmission containing the RPGT is presented. The mentioned transmission represents a conceptual design and includes two drives. The nomogram for the RPGT is shown in Figure 4. Normal (3) corresponds to the planet carrier, the outer normals (5) and (6) to the planet gears, and normals (1), (3), and (4) to central gears.

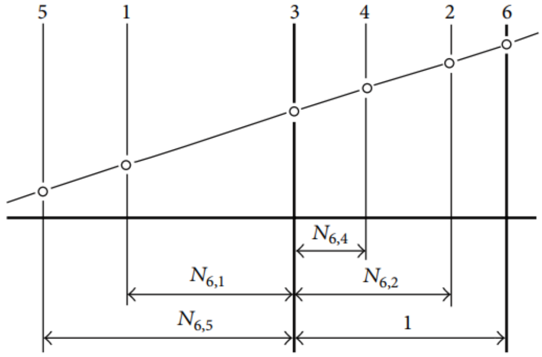


Fig. 4. Nomogram for RPGT [9]

The carrier normal (3) is placed at the zero position on the horizontal axis, and the outer planet gear (6) at a distance of 1 from normal (3), as shown in Figure 4. The distances of the other normal relative to the planet carrier normal represent the partial transmission ratios $N_{6,1}, N_{6,2}, N_{6,4}$ i $N_{6,5}$. Partial transmission ratios are defined using Willis's equation:

$$N_{gb,ga} = \frac{\omega_{ga} - \omega_c}{\omega_{gb} - \omega_c} = \frac{z_{gb}}{z_{ga}}, \quad (2)$$

where are: ω_{ga}, ω_{gb} and ω_c angular speeds of the input, output, and fixed elements, z_{gb} and z_{ga} the numbers of teeth of the output and input elements of the planetary gear train, respectively.

If a line is drawn at an angle across the normal, the distances of the intersected points on the normal relative to the horizontal define the angular speeds of the elements of the RPGT. Using the ratios of partial transmission on the horizontal axis and the differences in angular speeds on the vertical axis, the overall transmission ratios for each speed of the mentioned complex PGT are determined. Moreover, the advantage of the nomogram lies in drawing and determining the torque values on all elements of the RPGT, as well as listing all possible sequences of connecting the central elements with the clutches and brakes of the automatic transmission [10].

2.4 Discussion of Modern Methods

All three methods are based on graphical interpretation and analytical expression of transmission ratios. However, clear differences can be observed between them. The Lever Analogy Method provides precise results, provided that the distances between the nodal points on the lever are correctly defined. On the same lever, the values of peripheral speeds and torques of the elements of the RPGT can then be plotted in vector form, and their relationships can be further analyzed in terms of dynamic analysis. The procedure is very simple, but it is not suitable for computer implementation.

According to the Lever Method, [9, 10] it is possible to graphically and analytically represent the relationships between the angular speeds of the elements inside RPGT for each gear ratio using a nomogram. On the nomogram and the lever, it is possible to plot the values of peripheral speeds and torques, which can be used for dynamic analysis. The procedure for determining the transmission ratios is more complex when using the nomogram due to the methodology required to determine the transmission ratio values for each gear. A drawback of this method is that it requires additional use of Willis's equation and is not suitable for computer implementation.

The matrix method is the most complex of the analyzed methods, as it requires knowledge of a mathematics. Although it is based on graphical interpretation, the entire procedure is analytical and time-consuming. It is suitable for computer implementation due to the use of matrices and provides the most accurate results. The method can be easily adapted to changes in the design of a complex PGT, provided that the input parameters are correctly defined. This is not the case for the other two groups of methods.

3. CONCLUSIONS

The analyzed methods for determining the transmission ratios, i.e., the speeds of a complex planetary gear train, are complex to execute. Unlike conventional procedures, they provide more accurate results and better insight into the relationships between the gears and the planet carrier.

By applying these methods, any complex PGT can be analyzed, provided its elements are correctly represented graphically. In addition to kinematics, the methods allow for dynamic analysis of the PGT, as well as solving the problem of connecting the central elements to the input and output shafts of the automatic transmission via clutches. This provides a better understanding of the methodology for changing speeds and the loading of the elements of the PGT.

Each method has its own advantages and disadvantages, and their practical application depends on the initial requirements set during the design of an automatic transmission.

Among the analyzed methods, the use of graphs and the incidence matrix stands out due to their accuracy, easy adaptation to changes in the PGT design, and suitability for computer implementation.

Future research will focus on applying these methods to more complex PGT with 8, 9, and 10 speeds. In addition, the conditions for proper assembly and meshing of the elements of complex PGT, which are not fully addressed by these methods, will be thoroughly investigated.

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